Flanders Mathematical Olympiad 2000

Final Round

1. A natural number n consists of 7 different digits and is divisible by each of these digits. Which digits cannot occur in n?

2. Two triangles have the property that the sides of the second triangle are congruent to the medians of the first one. Find the ratio of the areas of these triangles.

3. Let p_n be the n-th prime ($p_1 = 2$). The sequence (f_n) is defined by $f_1 = 1$, $f_2 = 2$, and for each $j \geq 2$:

   (i) if $f_j = kp_n$ and $k < p_n$, then $f_{j+1} = (k+1)p_n$;

   (ii) if $f_j = p_n^2$ then $f_{j+1} = p_{n+1}$.

   (a) Prove that all terms of this sequence are distinct.

   (b) Find the position of the last term with less than three digits.

   (c) Determine the positive integers not occurring in the sequence.

   (d) How many numbers with less than three digits do occur in the sequence?

4. Find all x with $0 \leq x < 2\pi$ satisfying $\sin x < \cos x < \tan x < \cot x$.

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The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com