Flanders Mathematical Olympiad 1994

Final Round

1. In a right triangle, a is the length of the hypotenuse, and b and c the lengths of the legs. Find all real numbers x that satisfy $a^x > b^x + c^x$.

2. Determine all triples (a,b,c) of integers with $c \leq 94$ such that

$$(a+\sqrt{c})^2 + (b+\sqrt{c})^2 = 60 + 20\sqrt{c}.$$

3. Two different regular tetrahedrons A and B are contained in a cube of volume 1, with the vertices at vertices of the cube. Compute the volume of $A \cup B$.

4. The sequence of functions (f_n) is defined by $f_1(x) = x$ and $f_n(x) = \sqrt{f_{n-1}(x)} - \frac{1}{4}$ for $n \geq 2$.

    (a) Prove that $f_n(x) \leq f_{n-1}(x)$ for every x at which both functions are defined.

    (b) For each n, find the points x inside the domain for which $f_n(x) = x$.