21-st Brazilian Mathematical Olympiad 1999

Third Round

First Day

1. Let $ABCDE$ be a regular pentagon such that the star-like figure $ACEBD$ has area 1. Let P be the intersection of AC and BE and Q be the intersection of BD and CE. Determine the area of $APQD$.

2. Prove that there is at least one non-zero digit between the 1000000-th and the 3000000-th digit behind the comma in the decimal representation of $\sqrt{2}$.

3. Suppose that n pieces are placed on cells of a 10×10 board so that no four pieces form a rectangle with sides parallel to the sides of the board. Find the greatest possible value of n.

Second Day

4. On planet Zork there are some cities. For every city there is a city at the diametrically opposite point. Certain roads join the cities on Zork. If there is a road between cities P and Q, then there is also a road between the cities P' and Q' diametrically opposite to P and Q. Besides, the roads do not cross and for any two cities P and Q it is possible to travel from P to Q.

   The prices of *Kriptonita* in *Urghs* (the planetary currency) in two towns connected by a road differ by at most 100. Prove that there exist two diametrically opposite cities in which the prices of Kriptonita differ by at most 100 Urghs.

5. There are n football teams in Tumbólia. A championship is to be organized in which every two teams play exactly one match. All matches are played on Sundays, and no team plays more than one match in the same day. Determine the least positive integer m for which it is possible to set up a championship in m Sundays.

6. Given a triangle ABC, construct by ruler and compass a triangle $A'B'C'$ of the minimum area such that $C'\in CA$, $A'\in AB$, $B'\in BC$ and $\angle B'A'C'=\angle BAC$, $\angle A'B'C'=\angle ABC$.

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