Bulgarian Mathematical Olympiad 1978, III Round

First Day

1. There are given 10000 natural numbers forming an arithmetic progression which common difference is an odd number not divisible by 5. Prove that one of the given numbers ends with 1978 (1978 are last four digits in decimal counting system). (6 points, S. Dodunekov)

2. Circles c_1 and c_2 which radiuses are r_1 and r_2 respectively ($r_1 > r_2$) are tangent to each other internally. A line intersects circumferences in points A, B, C, D (the points are in this order on the line). Find the length of the segment AB if $AB : BC : CD = 1 : 2 : 3$ and the centers of the circumferences are on one and the same side of the line.
(7 points, G. Ganchev)

3. In the space are given n points in common position (there are no 4 points of them that are on one and the same plane). We observe all possible tetrahedrons all 4 vertices of which are from given points. Prove that if a plane contains no point from the given n points then the plane intersects at most

$$\frac{n^2(n-2)}{64}$$

from these tetrahedrons in quadrilaterals.
(7 points, N. Nenov, N. Hadzhiivanov)

Second day

4. Find all possible real values of p, q for which the solutions of the equation:

$$x^3 - px^2 + 11x - q = 0$$

are three consecutive whole numbers. (6 poins, Jordan Tabov)

5. It is given a pyramid which base is a rhombus $OABC$ with side length a. The edge is perpendicular to the base plane (containing $OABC$). From O are drawn perpendiculars OP, OQ, OS respectively to edges MA, MB, MC. Find the lengths of the diagonals of $OABC$ if it is known that OP, OQ, OS lies in one and the same plane. (7 points, G. Gantchev)

6. Prove that the number $\cos\frac{5\pi}{7}$ is:

   (a) root of the equation: $8x^3 - 4x^2 - 4x + 1 = 0$;

   (b) irrational number.

(7 points)

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