Bulgarian Mathematical Olympiad 1978, IV Round

First Day

1. It is given the sequence: $a_1, a_2, a_3, \ldots$, for which: $a_n = \frac{a_{n-1}^2 + c}{a_{n-2}}$ for all $n > 2$. Prove that the numbers a_1, a_2 and $\frac{a_1^2 + a_2^2 + c}{a_1 a_2}$ are whole numbers.
(6 points)

2. k_1 denotes one of the arcs formed by intersection of the circumference k and the chord AB. C is the middle point of k_1. On the half line (ray) PC is drawn the segment PM. Find the locus formed from the point M when P is moving on k_1.
(7 points, G. Ganchev)

3. On the name day of a men there are 5 people. The men observed that of any 3 people there are 2 that knows each other. Prove that the man may order his guests around circular table in such way that every man have on its both side people that he knows. (6 points, N. Nenov, N. Hazhiivanov)

Second day

4. Find greatest possible real value of S and smallest possible value of T such that for every triangle with sides a, b, c ($a \le b \le c$) to be true the inequalities:

$$S \le \frac{(a+b+c)^2}{bc} \le T$$

(7 points)

5. Prove that for every convex polygon can be found such a three sequential vertices for which a circle that they lies on covers the polygon.
(7 points, Jordan Tabov)

6. The base of the pyramid with vertex S is a pentagon $ABCDE$ for which $BC > DE$ and $AB > CD$. If AS is the longest edge of the pyramid prove that $BS > CS$. (7 points, Jordan Tabov)

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