10-th Canadian Mathematical Olympiad 1978

1. If for an integer n the tens digit of n^2 is 7, what is the units digit of n^2?

2. Find all pairs (a,b) of positive integers such that $2a^2 = 3b^3$.

3. Determine the largest possible value of z, if x, y, z are real numbers satisfying

$$x + y + z = 5,$$
$$xy + yz + zx = 3.$$

4. In a convex quadrilateral $ABCD$, lines AD and BC meet at E. Let H and G be the midpoints of BD and AC, respectively. Find the ratio of the area of triangle EHG to that of the quadrilateral $ABCD$.

5. Eve and Odette play a game on a 3×3 checkerboard with black and white checkers. The rules are as follows:

   (i) They play alternately.

   (ii) A turn consists of placing a checker on an unoccupied square of the board.

   (iii) A player in turn may select a checker of any color.

   (iv) When the board is full, Eve obtains one point for every row, column or diagonal with an even number of black checkers, and Odette obtains one point for every row, column or diagonal with an odd number of black checkers.

   (v) The player obtaining at least five of the eight points wins.

   (a) Is a $4:4$ tie possible?

   (b) Describe a winning strategy for the girl who plays first.

6. Sketch the graph of $x^3 + xy + y^3 = 3$.