

13-th Canadian Mathematical Olympiad 1981

May 6, 1981

1. Show that the following equation has no real solution:

$$[x] + [2x] + [4x] + [8x] + [16x] + [32x] = 12345.$$

2. A line l is tangent to a circle of radius r at a point P . From a variable point R on the circle, a perpendicular RQ is drawn to l with $Q \in l$. Determine the maximum area of triangle PQR .
3. Given a finite set of lines in a plane P , show that it is possible to draw an arbitrarily large circle in P which does not meet any of them. On the other hand, show that it is possible to arrange an infinite sequence of lines in P so that every circle in P (of nonzero radius) meets at least one of the lines.
4. Suppose that two polynomials $P(x)$ and $Q(x)$ satisfy $P(Q(x)) = Q(P(x))$ for all real x . If the equation $P(x) = Q(x)$ has no real solution, show that the equation $P(P(x)) = Q(Q(x))$ also has no real solution.
5. Eleven theatrical groups participated in a festival. Each day, some groups were scheduled to perform while the remaining groups joined the general audience. At the conclusion of the festival, each group had seen, during its days off, at least one performance of every other group. At least how many days did the festival last?