20-th Canadian Mathematical Olympiad 1988

March 30, 1988

1. Find all values of b for which the equations $1988x^2 + bx + 8891 = 0$ and $8891x^2 + bx + 1988 = 0$ have a common root.

2. A house has the shape of a triangle of permieter P meters and area A square meters. The garden consists of all the land within 5 meters of the house. How much land do the garden and house together occupy?

3. Let $\mathscr{S}$ be a finite set of at least five points in the plane, some of which are colored red and the others are colored blue. No three points of the same color are collinear. Show that there is a triangle with the vertices in $\mathscr{S}$ such that

   (i) all its vertices are the same color, and

   (ii) at least one side of the triangle contains no point of the opposite color.

4. Define $x_0 = 0$, $x_1 = 1$, $x_{n+1} = 4x_n - x_{n-1}$, and $y_0 = 1$, $y_1 = 2$, $y_{n+1} = 4y_n - y_{n-1}$. Prove that $y_n^2 = 3x_n^2 + 1$ for all $n \geq 0$.

5. For every set A of integers, $p(A)$ denotes the product of the elements of A. Also, for a set S of integers, $m(S)$ denotes the arithmetic mean of $p(A)$ over all nonempty subsets A of S. Let $S = \{a_1, \dots, a_r\}$ be a set of integers. If $m(S) = 13$ and $m(S \cup \{a_{r+1}\}) = 49$ for some positive integer a_{r+1}, determine the values of $a_1, \dots, a_r$ and a_{r+1}.

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