Croatian Team Selection Test 2001

Makarska, May 11

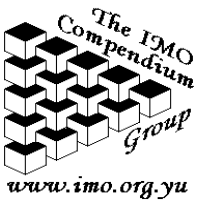
1. Consider $A = \{1, 2, \ldots, 16\}$. A partition of A into nonempty sets $A_1, A_2, \ldots, A_n$ is said to be *good* if none of the A_i contains elements a, b, c (not necessarily distinct) such that $a = b + c$.

   (a) Find a good partition $\{A_1, A_2, A_3, A_4\}$ of A.

   (b) Prove that no partition $\{A_1, A_2, A_3\}$ of A is good.

2. Circles k_1 and k_2 intersect at P and Q, and A and B are the tangency points of their common tangent that is closer to P (where A is on k_1 and B on k_2). The tangent to k_1 at P intersects k_2 again at C. The lines AP and BC meet at R. Show that the lines BP and BC are tangent to the circumcircle of triangle PQR.

3. Find all solutions of the equation $(a^a)^5 = b^b$ in positive integers.



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