Croatian Team Selection Test 2004

Trogir, May 7

1. Find all pairs (x,y) of positive integers such that $x(x+y) = y^2 + 1$.

2. Prove that if a,b,c are positive numbers with $abc = 1$, then

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq a + b + c.$$

3. A line intersects a semicircle with diameter AB and center O at C and D, and the line AB at M, where $MB < MA$ and $MD < MC$. If the circumcircles of the triangles AOC and DOB meet again at K, prove that $\angle MKO$ is right.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com