Croatian Team Selection Test 2006

April 27

1. Find all natural numbers that can be expressed in a unique way as a sum of five or less perfect squares.

2. Assume that a, b, and c are positive real numbers for which $(a+b)(a+c)(b+c)=1$. Prove that $ab+bc+ca\leq\frac{3}{4}$.

3. Let ABC be a triangle for which $|AB|+|BC|=3|AC|$. Let D and E be the points of tangency of the incircle with the sides AB and BC respectively, and let K and L be the other endpoints of the diameters originating from D and E. Prove that C, A, L, and K lie on a circle.

4. Find all natural solutions of: $3^x=2^xy+1$.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com