Croatian Team Selection Test 2008

Primošten, April 8

1. Let x,y,z be positive numbers. Find the minimum value of

$$(a)\ \frac{x^2+y^2+z^2}{xy+yz}, \qquad (b)\frac{x^2+y^2+2z^2}{xy+yz}.$$

2. For which natural numbers n do there exist rational numbers a and b which are not integers such that both $a+b$ and a^n+b^n are integers?

3. Point M is taken on side BC of a triangle ABC such that the centroid T_c of triangle ABM lies on the circumcircle of $\triangle ACM$ and the centroid T_b of $\triangle ACM$ lies on the circumcircle of $\triangle ABM$. Prove that the medians of the triangles ABM and ACM from M are of the same length.

4. Let S be the set of all odd positive integers less than $30m$ which are not multiples of 5, where m is a given positive integer. Find the smallest positive integer k such that each k-element subset of S contains two distinct numbers, one of which divides the other.

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