42-nd Czech and Slovak Mathematical Olympiad 1993

1. Find all natural numbers n for which $7^n - 1$ is divisible by $6^n - 1$.

2. In fields of a 19×19 table are written integers so that any two lying on neighboring fields differ at most by 2 (two fields are neighboring if they share a side). Find the greatest possible number of mutually different integers in such a table.

3. Let AKL be a triangle such that $\angle ALK > 90^\circ + \angle LAK$. Construct an equilateral trapezoid $ABCD$ (i.e. with three sides equal) with $AB \perp CD$ such that K lies on the side BC, L on the diagonal AC and the lines AK and BL intersect at the circumcenter of the trapezoid.

4. The sequence (a_n) of natural numbers is defined by $a_1 = 2$ and a_{n+1} equals the sum of tenth powers of the digits of a_n for all $n \geq 1$. Are there numbers which appear twice in the sequence (a_n)?

5. Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(-1) = f(1)$ and
$$f(x) + f(y) = f(x + 2xy) + f(y - 2xy) \quad \text{for all } x, y \in \mathbb{Z}.$$

6. Show that there exists a tetrahedron which can be partitioned into eight congruent tetrahedra, each of which is similar to the original one.

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