

37-th German Federal Mathematical Competition 2006/07

First Round

1. The vertices and the midpoints of the sides of a given regular 2007-gon are to be numbered with numbers $1, 2, \dots, 4014$ so that the sum of the three numbers at every side is the same. Show that such a numbering is possible.
2. Each positive integer is colored red or green in such a way that the sum of any three (not necessarily different) equally colored numbers also has the same color. Find all possible colorings.
3. Points E and F are taken on the sides AC and BC respectively of a triangle ABC such that $AE = BF$. The circles passing through A, C, F and through B, C, E intersect again at point D . Prove that the line CD bisects the angle ACB .
4. Given a positive integer a , how many nonnegative integer solutions does the equation $\left\lfloor \frac{x}{a} \right\rfloor = \left\lfloor \frac{x}{a+1} \right\rfloor$ have?