1-st German Federal Mathematical Competition 1970/71

First Round

1. The numbers $1,2,\dots,1970$ are written on a board. It is allowed to erase two numbers and write down their difference instead. This operation is repeated until only one number remains. Prove that this number is odd.

2. We are given a piece of paper. We cut it into 8 or 12 arbitrary pieces, then we choose one of the obtained pieces and either cut it into 8 or 12 pieces or do not cut it, etc. Can we obtain exactly 60 pieces in this way? Prove that every number of pieces greater than 60 can be obtained.

3. Suppose five segments are given such that any three of them are sides of a triangle. Prove that some three of the segments are sides of an acute-angled triangle.

4. Let P and Q be two horizontally neighboring fields of an $n \times n$ chessboard, P being to the left of Q. A piece standing on field P is to be moved around the chessboard. In each move, the piece can be moved to the neighboring field to the right, up, or down-left. (For instance, it can be moved from e5 to e6, f5 or d4.) Prove that for no value of n can the piece visit every field of the chessboard exactly once and end up on field Q.

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