2-nd German Federal Mathematical Competition 1971/72

First Round

1. In each cell of an $n \times n$ board there is a real number. Assume that the sum of the numbers in each *cross* (formed by a row and a column) is at least A. Find the smallest possible sum of all numbers in the board.

2. Let $n \geq 3$ equal round beer coasters $B_1, B_2, \ldots, B_n$ be placed on a flat table so that B_i touches B_{i+1} for $i = 1, 2, \ldots, n$ (where $B_{n+1} = B_1$). Another beer coaster B rolls along the exterior boundary of the chain of coasters. How many revolutions will coaster B make before reaching its initial position?

3. Suppose 2^{n-1} subsets of an n-element set are selected in such a way that any three of them have an element in common. Prove that all the subsets have a common element.

4. Show that, when n goes through all natural numbers, the sequence $\left[n + \sqrt{n} + \frac{1}{2}\right]$ contains all natural numbers except for the perfect squares.

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