

3-rd German Federal Mathematical Competition 1972/73

First Round

1. A natural number has 1000 decimal digits, at most one of which is different from 5. Prove that this number is not a perfect square.
2. From each of the points A and B on a flat lake one can reach any point on the coast by a straightforward travel. Show that one can also reach every point on the coast by a straightforward travel starting from any point on the segment AB .
3. Let be given n arbitrary digits a_1, \dots, a_n . Does there exist a natural number such that the decimal digits of its square root after the decimal point are exactly these n digits in that order? Justify your answer.
4. There are n people sitting at a round table. Suppose that the number of persons whose neighbors to the right are of the same sex is the same as the number of persons for which this does not hold. Prove that n is divisible by 4.