8-th German Federal Mathematical Competition 1977/78

First Round

1. The knight-piece from the game of chess is modified so that it moves p fields horizontally or vertically and q fields in the perpendicular direction. Also assume that the chessboard is infinite. If the knight returns to the initial field after n moves, show that n must be even.

2. A set of n^2 counters are labelled with $1,2,\dots,n$, each label appearing n times. Can one arrange the counters on a line in such a way that for all $x \in \{1,2,\dots,n\}$, between any two successive counters with the label x there are exactly x counters (with labels different from x)?

3. For every positive integer n, define the *remainder sum* $r(n)$ as the sum of the remainders upon division of n by each of the numbers 1 through n. Prove that if the larger of two consecutive integers is a power of 2, then these two numbers have the same remainder sum.

4. In a triangle ABC, A_1,B_1,C_1 are symmetric to A,B,C with respect to B,C,A, respectively. Given the points A_1,B_1,C_1, construct the triangle ABC.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com