9-th German Federal Mathematical Competition 1978/79

First Round

1. There are n teams in a football league. During a championship every two teams play exactly one match, but no team can play more than one match in a week. At least, how many weeks are necessary for the championship to be held? Give an schedule for such a championship.

2. Let $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$ be equally oriented squares in a plane. Assume that $A_1 = A_2$. Prove that the lines B_1B_2, C_1C_2, and D_1D_2 have a point in common.

3. In base 10 there exist two-digit natural numbers that can be factorized into two natural factors such that the two digits and the two factors of each number form a sequence of four consecutive integers (for example, $12 = 3 \cdot 4$). Determine all such numbers in all bases.

4. Let a be an integer not divisible by 5. Prove that the polynomial $f(x) = x^5 - x + a$ cannot be written as a product of two non-constant polynomials with integer coefficients.

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