12-th German Federal Mathematical Competition 1981/82

First Round

1. Let S be the sum of the greatest odd divisors of the natural numbers 1 through 2^n. Prove that $3S = 4^n + 2$.

2. In a convex quadrilateral $ABCD$ sides AB and DC are both divided into m equal parts by points $A, S_1, S_2, \dots, S_{m-1}, B$, and $D, T_1, T_2, \dots, T_{m-1}, C$, respectively (in this order). Similarly, sides BC and AD are divided into n equal parts by points $B, U_1, U_2, \dots, U_{n-1}, C$, and $A, V_1, V_2, \dots, V_{n-1}, D$. Prove that each of the segments S_iT_i is divided by the segments U_jV_j $(1 \le j \le n-1)$ into n equal parts.

3. A convex 1982-gon is given in the plane. Consider all triangles with the vertices at vertices of the polygon. A point P inside the polygon does not lie on any of the triangles' sides. Prove that the number of the considered triangles that contain point P is even.

4. A set of real numbers is called *sum-free* if there are no three elements x, y, z of the set satisfying $x + y = z$. A sum-free subset of the set $\{1, 2, \dots, 2n+1\}$ has k elements. Find the largest possible value of k.

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