20-th German Federal Mathematical Competition 1989/90

First Round

1. Consider the trinomial $f(x) = x^2 + 2bx + c$ with integer coefficients b and c. Prove that if $f(n) \geq 0$ for all integers n, then $f(x) \geq 0$ even for all rational numbers x.

2. The sequence $a_0, a_1, a_2, \ldots$ is defined by $a_0 = 0$, $a_1 = a_2 = 1$ and

$$a_{n+2} + a_{n-1} = 2(a_{n+1} + a_n) \quad \text{for all } n \in \mathbb{N}.$$

   Show that all a_n are perfect squares.

3. There are 172 two-way direct airways between 20 cities, at most one between any two cities. Prove that one can reach any city from any other city with at most one transfer.

4. Suppose that every two opposite edges of a tetrahedron are orthogonal. Show that the midpoints of the six edges lie on a sphere.

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