35-th German Federal Mathematical Competition 2004/05

Second Round

1. Players A and B are playing a game on a 100×100 chessboard. In the beginning A has a marker on the lower left square whereas B has a marker on the lower right square. With A starting, the players alternately move their marker horizontally or vertically to an adjacent square. Prove that A can always play so that after finitely many moves his marker lies on the same square as B's marker.

2. A rational number x is given. Show that there are only finitely many triples (a,b,c) of integers with $a < 0$ and $b^2 - 4ac = 5$ for which $ax^2 + bx + c > 0$.

3. Circles k_1 and k_2 meet at two distinct points A and B. Two lines through B meet k_1 at C and D and k_2 at E and F, respectively, so that B lies on the segments CE and DF. Let M be the midpoint of CE and N that of DF. Prove that the triangles ACD, AEF, and AMN are similar.

4. We consider closed polygonal lines $P_1P_2 \dots P_nP_1$ ($n \geq 3$), where no two of the n points $P_1, \dots, P_n$ are collinear. Let $A(n)$ be the largest possible number of self-intersections of such a polygonal line. Prove that $A(n) = \frac{n(n-3)}{2}$ for n odd and $A(n) = \frac{n(n-4)}{2} + 1$ for n even.

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