

# 3-rd German Federal Mathematical Competition 1972/73

## Second Round

1. Let be given 51 points in a square of side 7. Prove that among these points there always exist three that lie inside a circle with radius 1.
2. The following operations on a natural number are permitted:
  - (i) write digit 4 at the end of its decimal representation;
  - (ii) write digit 0 at the end of its decimal representation;
  - (iii) divide it by 2 if it is even.

Show that, starting with number 4, we can obtain every positive integer using finitely many operations (i), (ii), (iii).

3. The floor of a rectangular room can be tiled with rectangular tiles  $2 \times 2$  and  $4 \times 1$ . Prove that if we replace one tile with a tile of the other type, then a tiling will no longer be possible.
4. Prove that for every positive integer  $n$  there is a natural number whose base 10 representation consists only of digits 1 and 2, and that is divisible by  $2^n$ . Is the statement true in bases 4 and 6?