5-th German Federal Mathematical Competition 1974/75

Second Round

1. Let a,b,c,d be distinct positive real numbers. Prove that if one of the numbers c,d lies between a and b, or one of a,b lies between c and d, then

$$\sqrt{(a+b)(c+d)} > \sqrt{ab} + \sqrt{cd},$$

and that otherwise one can choose a,b,c,d so that this inequality is false.

2. Prove that no term of the sequence $10001, 100010001, 1000100010001, \ldots$ is prime.

3. For n positive integers $x_1, x_2, \ldots, x_n$, a_n is their arithmetic and g_n the geometric mean. Consider the statement S_n: If a_n/g_n is a positive integer, then $x_1 = x_2 = \cdots = x_n$. Prove S_2 and disprove S_n for all even $n > 2$.

4. Two brothers inherited n gold pieces of the total weight $2n$. The weights of the pieces are integers, and the heaviest piece is not heavier than all the other pieces together. Show that if n is even, the brother can divide the inheritance into two parts of equal weight.

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