

# 12-th German Federal Mathematical Competition 1981/82

## Second Round

1. Max divided a natural number  $p$  by a natural number  $q \leq 100$ . In the decimal representation of the quotient he calculated, the sequence of digits 1982 occurs somewhere after the decimal point. Show that Max made a computational mistake.
2. Decide whether every triangle  $ABC$  in space can be orthogonally projected onto a plane such that the projection is an equilateral triangle  $A'B'C'$ .
3. Let  $a_1, a_2, \dots, a_n$  be nonnegative real numbers with  $a_1 + \dots + a_n = 1$ . Prove that the expression

$$\frac{a_1}{1 + a_2 + \dots + a_n} + \frac{a_2}{1 + a_1 + a_3 + \dots + a_n} + \dots + \frac{a_n}{1 + a_1 + \dots + a_{n-1}}$$

attains its minimum, and determine this minimum.

4. A positive integer  $n$  is such that  $4^n + 2^n + 1$  is prime. Prove that  $n$  is a power of three.