

20-th German Federal Mathematical Competition 1989/90

Second Round

1. Find all triples (a, b, c) of positive integers such that the product of any two of them when divided by the third leaves the remainder 1.
2. Let $A(n)$ be the least possible number of distinct points in the plane with the following property: For every $k = 1, 2, \dots, n$ there is a line containing precisely k of these points. Show that $A(n) = \left\lceil \frac{n+1}{2} \right\rceil \left\lceil \frac{n+2}{2} \right\rceil$.
3. Given any five nonnegative real numbers with the sum 1, show that they can be arranged around a circle in such a way that the five products of two consecutive numbers sum up to at most $1/5$.
4. A worm of length 1 lies on a plane. Show that it can be covered by a semicircle with diameter 1.