12-th Hellenic Mathematical Olympiad 1995

Seniors

1. Find all positive integers n such that $-5^4+5^5+5^n$ is a perfect square. Do the same for $2^4+2^7+2^n$.

2. Let ABC be a triangle with $AB=AC$ and let D be a point on BC such that the incircle of $\triangle ABD$ and the excircle of $\triangle ADC$ corresponding to A have the same radius. Prove that this radius is equal to one quarter of the altitude from B of triangle ABC.

3. If the equation $ax^2+(c-b)x+(e-d)=0$ has real roots greater than 1, prove that the equation $ax^4+bx^3+cx^2+dx+e=0$ has at least one real root.

4. Let be given lines $l_1,l_2,\ldots,l_k$ in the plane, no two of which are parallel and no three of which are concurrent. For which k can one label the intersection points of these lines by $1,2,\ldots,k-1$ so that in each of the given lines all the labels appear exactly once?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com