16-th Hellenic Mathematical Olympiad 1999

Seniors

1. Let $f(x) = ax^2 + bx + c$, where a, b, c are nonnegative real numbers, not all equal to zero. Prove that $f(xy)^2 \leq f(x^2) f(y^2)$ for all real numbers x, y.

2. A right triangle has integer side lengths, and the sum of its area and the length of one of its legs equals 75. Find the side lengths of the triangle.

3. In an acute-angled triangle ABC, AD, BE and CZ are the altitudes and H the orthocenter. Lines EZ and BC meet at N. The line passing through D and parallel to ZE meets lines AB and AC at K and L, respectively. Prove that the circumcircle of the triangle NKL bisects the side BC.

4. On a circle are given $n \geq 3$ points. At most, how many parts can the segments with the endpoints at these n points divide the interior of the circle into?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com