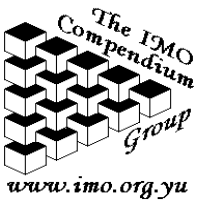
Eötvös Mathematical Competition 1901

1. Prove that, for any positive integer n, $1^n + 2^n + 3^n + 4^n$ is divisible by 5 if and only if n is not divisible by 4.

2. If $u = \cot 22°30'$ and $v = 1/\sin 22°30'$, prove that u satisfies a quadratic and v a fourth degree equation with integral coefficients and leading coefficient 1.

3. Let a and b two natural numbers whose greatest common divisor is d. Prove that exactly d of the numbers $a, 2a, 3a, \ldots, ba$ are divisible by b.



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