Eötvös Mathematical Competition 1906

1. Prove that if $\tan\frac{\alpha}{2}$ is rational (or undefined) then so are $\cos\alpha$ and $\sin\alpha$; Conversely, if $\cos\alpha$ and $\sin\alpha$ are rational then $\tan\frac{\alpha}{2}$ is rational or undefined.

2. Let K, L, M, N be the centers of the squares erected externally on the sides of a rhombus. Prove that $KLMN$ is a square.

3. Let $a_1, a_2, \ldots, a_n$ be an arbitrary arrangement of the numbers $1, 2, \ldots, n$. If n is odd, prove that the product $(a_1 - 1)(a_2 - 2)\cdots(a_n - n)$ is even.

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