Eötvös Mathematical Competition 1908

1. If a and b are odd integers, prove that $a^3 - b^3$ is divisible by 2^n if and only if so is $a - b$.

2. Let $n > 2$ be an integer. Prove that the n-th power of the length of the hypotenuse of a right triangle is greater than the sum of the n-th powers of the lengths of the legs.

3. A circumference is divided into 10 equal arcs. Join each division point (1) to the next by straight line segments, and then (2) to the next but two by straight line segments (see the figures). Prove that the difference in the side lengths of the obtained decagons is equal to the radius of their circumcircle.

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