Eötvös Mathematical Competition 1909

1. Consider any three consecutive natural numbers. Prove that the cube of the largest number cannot be the sum of the cubes of the other two.

2. Show that the radian measure of an acute angle is less than the arithmetic mean of its sine and its tangent.

3. Let A_1, B_1, C_1 be the feet of the altitudes of $\triangle ABC$ from A, B and C respectively, and let M be the orthocenter. Assume that the orthic triangle $A_1B_1C_1$ is non-degenerate. Prove that each of the points M, A, B and C is the center of a circle tangent to the three sides (extended if necessary) of $\triangle A_1B_1C_1$. What is the difference in the behavior of acute and obtuse triangles ABC?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu
Typed in LaTeX by Ercole Suppa