Eötvös Mathematical Competition 1910

1. If real numbers a, b, c satisfy $a^2 + b^2 + c^2 = 1$, prove the inequalities
$$-\frac{1}{2} \leq ab + bc + ca \leq 1.$$

2. Let a, b, c, d and u be integers such that each of the numbers ac, $bc + ad$, bd is a multiple of u, show that bc and ad also are multiples of u.

3. The lengths of sides CB and CA of $\triangle ABC$ are a and b, and the angle between them is $\gamma = 120^\circ$. Express the length of the bisector of γ in terms of a and b.

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