Kürschák Mathematical Competition 1983

1. A cube of integer edge lengths is given in space so that all four vertices of one of the faces are lattice points. Prove that the other four vertices are also lattice points.

2. Prove that for any integer $k > 2$ there exist infinitely many positive integers n such that the least common multiple of $n, n+1, \ldots, n+k-1$ is greater than the least common multiple of $n+1, n+2, \ldots, n+k$.

3. Each integer is colored in one of 100 colors so that all the colors are used. Also assume that for any choice of intervals $[a,b]$ and $[c,d]$ of equal length and with integral endpoints, if a and c have the same color and so do b and d, then the entire intervals $[a,b]$ and $[c,d]$ are identically colored (i.e. for any integer x with $0 \leq x \leq b-a$ the numbers $a+x$ and $c+x$ are of the same color). Prove that -1982 and 1982 are of different colors.

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