Kürschák Mathematical Competition 1986

1. The convex $(n+1)$-gon $P_0P_1\dots P_n$ is partitioned into triangles by $n-2$ nonintersecting diagonals. Prove that the triangles can be numbered from 1 to $n-1$ so that for P_i is a vertex of the triangle i for each $1 \le i \le n-1$.

2. Let n be a positive integer. For each prime divisor p of n, consider the highest power of p which does not exceed n. The sum of these powers is defined as the power-sum of n. Prove that there exist infinitely many positive integers which are greater than their respective power-sums.

3. Each vertex of a triangle is reflected in the opposite side. Prove that the area of the triangle determined by the three points of reflection is less than five times the area of the original triangle.

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