Eötvös Mathematical Competition 1897

1. If α, β, γ are the angles of a right triangle, prove the relation:

$$\sin\alpha\sin\beta\sin(\alpha-\beta)+\sin\beta\sin\gamma\sin(\beta-\gamma)+\sin\gamma\sin\alpha\sin(\gamma-\alpha)+ \\ +\sin(\alpha-\beta)\sin(\beta-\gamma)+\sin(\gamma-\alpha)=0$$

2. Show that if α, β and γ are the angles of an arbitrary triangle, then

$$\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}<\frac{1}{4}.$$

3. A line e intersects the sides AB, CD, AD and BC (or their extensions) at points M, N, P, Q, respectively. Given the points M, N, P, Q and the length p of side AB, construct the rectangle. Under what conditions can this problem be solved, and how many solutions does it have?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu
Typed in LaTeX by Ercole Suppa