Eötvös Mathematical Competition 1899

1. The points A_0, A_1, A_2, A_3, A_4 divide a unit circle into five equal parts. Prove that the chords A_0A_1 and A_0A_2 satisfy $(A_0A_1 \cdot A_0A_2)^2 = 5$.

2. If x_1 and x_2 are the roots of the equation $x^2 - (a+d)x + ad - bc = 0$, show that x_1^3 and x_2^3 are the roots of

$$y^2 - \left(a^3 + d^3 + 3abc + 3bcd\right) y + (ad - bc)^3 = 0.$$

3. Prove that $A = 2903^n - 803^n - 464^n + 261^n$ is divisible by 1897 for any natural number n.

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imo.org.yu
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