# 10-th Indian Mathematical Olympiad 1995

1. In an acute-angled triangle $ABC$ with $\angle A = 30^\circ$, $H$ is the orthocenter and $M$ the midpoint of $BC$. Point $T$ is symmetric to $H$ with respect to $M$. Show that $AT = 2BC$.

2. Show that there are infinitely many pairs $(a,b)$ of coprime integers such that both the quadratic equations $x^2 + ax + b = 0$ and $x^2 + 2ax + b = 0$ have integer roots.

3. Show that the number of three-element subsets $\{a,b,c\}$ of $\{1,2,\dots,63\}$ with $a+b+c < 95$ is less than the number of those with $a+b+c > 95$.

4. Let $\Gamma'$ be the circle lying inside a triangle $ABC$ and touching the sides $AB$ and $AC$ and the incircle $\Gamma$ of the triangle externally. Show that the ratio of the radii of the circles $\Gamma'$ and $\Gamma$ equals $\tan^2 \frac{\pi-\alpha}{4}$.

5. The real numbers $a_1, a_2, \dots, a_n$ are all greater than 1 and satisfy $|a_k - a_{k+1}| < 1$ for $1 \le k \le n-1$. Prove that
$$\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} < 2n - 1.$$

6. Find all primes $p$ for which $\dfrac{2^{p-1}-1}{p}$ is a perfect square.



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