18-th Italian Mathematical Olympiad 2002

Cesenatico, May 3, 2002

1. Find all three-digit numbers which are equal to 34 times the sum of their digits.

2. A house has the shape of a letter L obtained by placing side by side, in a suitable way, four squares of side length 10 meters. The six sidewalls are all 10 meters high and the roof has six faces inclined by the angle of $30°$ with respect to the horizontal plane. Calculate the volume of the house.

3. A line r and points A and B not on r are given in the plane. Let R and S be the feet of perpendiculars from A and B to r respectively, and M be the midpoint of AB. Prove that the circumcircles of triangles ARM and BSM are congruent (assuming that these triangles are non-degenerate).

4. Determine the values of n for which all the solutions of the equation $x^3 - 3x + n = 0$ are integers.

5. Prove that if n is a positive integer such that $m = 5^n + 3^n + 1$ is prime, then n is divisible by 12.

6. Let be given a 100×100 chessboard whose all cels are empty.

   (a) Is it possible to color an odd number of cells so that every colored cell is adjacent to an odd number of colored cells?

   (b) Is it possible to color some cells so that an odd number of them are adjacent to exactly four, while the remaining colored cells are adjacent to exactly two colored cells?

   (c) Is it possible to color some cells so that an odd number of them are adjacent to exactly two, and the remaining colored cells are adjacent to exactly four colored cells?

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