

12-th Italian Mathematical Olympiad 1996

Cesenatico, May 3, 1996

1. Among the triangles with one side l and the area S , find those with the maximum possible product of the three altitudes.
2. Prove that the equation $a^2 + b^2 = c^2 + 3$ has infinitely many integer solutions (a, b, c) .
3. Let A and B be opposite vertices of a unit cube. Find the radius of the sphere with center inside the cube, tangent to the three faces meeting at A and to the three edges meeting at B .
4. Find the number of words of n letters from the alphabet $\{a, b, c\}$ which contain an even number of a 's.
5. Let A be a point outside a circle \mathcal{C} . For each point P on \mathcal{C} , a square $APQR$ is constructed, with the vertices A, P, Q, R in counterclockwise order. Find the locus of Q as P runs over \mathcal{C} .
6. What is the minimum number of squares that one needs to draw on a blank sheet in order to obtain a complete grid $n \times n$?

Time allowed: 4.5 hours