

13-th Italian Mathematical Olympiad 1997

Cesenatico, May 2, 1997

1. An infinite rectangular stripe of width 3cm is folded along a line. What is the minimum possible area of the region of overlapping?
2. Let a real function f defined on the real numbers satisfy the following conditions:
 - (i) $f(10+x) = f(10-x)$;
 - (ii) $f(20+x) = -f(20-x)$for all x . Prove that f is odd and periodic.
3. The positive quadrant of a coordinate plane is divided into unit squares by lattice lines. Is it possible to color the squares in black and white so that:
 - (i) In every square of side n ($n \in \mathbb{N}$) with a vertex at the origin and sides are parallel to the axes, there are more black than white squares;
 - (ii) Every diagonal parallel to the line $y = x$ intersects only finitely many black squares?
4. In a tetrahedron $ABCD$, let a and S be respectively the length of side AB and the area of the projection of the tetrahedron to the line perpendicular to AB . Determine the volume of the tetrahedron.
5. Let X be the set of natural numbers whose all digits in the decimal representation are different. For $n \in \mathbb{N}$, denote by A_n the set of numbers whose digits are a permutation of the digits of n , and d_n be the greatest common divisor of the numbers in A_n . (For example, $A_{1120} = \{112, 121, \dots, 2101, 2110\}$, so $d_{1120} = 1$.) Find the maximum possible value of d_n .
6. A tourist wants to visit each of the ten cities shown on the picture. The continuous segments on the picture denote railway lines, whereas the dashed segments denote air lines. A railway line costs 150000 liras, and an air line costs 250000 liras. What is the minimum possible price of a desired route?