Italian IMO Team Selection Test 1994

Cortona, June 11, 1994

Time allowed: 4 hours

1. Given a circle γ and a point P inside it, find the maximum and minimum value of the sum of the lengths of two perpendicular chords of γ passing through P.

2. Find all prime numbers p for which $\dfrac{2^{p-1}-1}{p}$ is a perfect square.

3. Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the condition
$$f(x - f(y)) = 1 + x - y \quad \text{for all } x, y \in \mathbb{R}.$$

4. Let X be a set of n elements and k be a positive integer. Consider the family $\mathscr{E}_k$ of all k-tuples $(E_1, \ldots, E_k)$ with $E_i \subseteq X$ for each i. Evaluate the sums
$$\sum_{(E_1,\ldots,E_k)\in\mathscr{E}_k} |E_1 \cap \cdots \cap E_k| \quad \text{and} \quad \sum_{(E_1,\ldots,E_k)\in\mathscr{E}_k} |E_1 \cup \cdots \cup E_k|.$$