Italian IMO Team Selection Test 1996

Cortona, June 8, 1996

Time allowed: 4 hours

1. Let A and B be two diametrically opposite points on a circle with radius 1. Points $P_1, P_2, \ldots, P_n$ are arbitrarily chosen on the circle. Let a and b be the geometric means of the distances of $P_1, P_2, \ldots, P_n$ from A and B, respectively. Show that at least one of the numbers a and b does not exceed $\sqrt{2}$.

2. Let $A_1, A_2, \ldots, A_n$ be distinct subsets of an n-element set X ($n \geq 2$). Show that there exists an element x of X such that the sets $A_1 \setminus \{x\}, \ldots, A_n \setminus \{x\}$ are all distinct.

3. Let $ABCD$ be a parallelogram with side AB longer than AD and acute angle $\angle DAB$. The bisector of $\angle DAB$ meets side CD at L and line BC at K. If O is the circumcenter of triangle LCK, prove that the points B, C, O, D lie on a circle.

4. Prove that there exists a set X of 1996 positive integers with the following properties:

   (i) the elements of X are pairwise coprime;

   (ii) all elements of X and all sums of two or more distinct elements of X are composite numbers.

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