13-th Nordic Mathematical Contest

April 12, 1999

1. The function f is defined for nonnegative integers and satisfies

$$f(n) = \begin{cases} f(f(n+11)) & \text{if } n \leq 1999, \\ n-5 & \text{if } n > 1999. \end{cases}$$

   Find all solutions of the equation $f(n) = 1999$.

2. A convex heptagon with all different sides is inscribed in a circle. At most, how many angles equal to 120° can this heptagon have?

3. Nonnegative integers a and b are given. A soldier is walking on the infinite lattice $\mathbb{Z} \times \mathbb{Z}$ as follows. In each step, from a point (x,y) he is only allowed to go to one of the points $(x \pm a, y \pm b)$ and $(x \pm b, y \pm a)$. Find all values of a and b for which the soldier can visit every point of the lattice during his infinite walk.

4. Let $a_1, a_2, \dots, a_n$ be positive numbers ($n \geq 1$). Show that

$$n\left(\frac{1}{a_1} + \cdots + \frac{1}{a_n}\right) \geq \left(\frac{1}{1+a_1} + \cdots + \frac{1}{1+a_n}\right)\left(n + \frac{1}{a_1} + \cdots + \frac{1}{a_n}\right).$$

   When does equality hold?

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