15-th Nordic Mathematical Contest

March 29, 2001

1. Let A be a finite set of unit squares in the coordinate plane, each of which has vertices at integer points. Show that there exists a subset B of A consisting of at least $1/4$ of the squares in A such that no two distinct squares in B have a common vertex.

2. A function $f:\mathbb{R}\to\mathbb{R}$ is bounded and satisfies
$$f\left(x+\frac{1}{3}\right)+f\left(x+\frac{1}{2}\right)=(x)+f\left(x+\frac{5}{6}\right)$$
for all real x. Show that f is periodic.

3. Find the number of real roots of the equation
$$x^8-x^7+2x^6-2x^5+3x^4-3x^3+4x^2-4x+\frac{5}{2}=0.$$

4. Each of the diagonals AD, BE and CF of a convex hexagon $ABCDEF$ divides its area into two equal parts. Prove that these three diagonals pass through the same point.

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