48-th Polish Mathematical Olympiad 1996/97

Second Round

February 21–22, 1997

First Day

1. For any real number a find the number of ordered triples (x,y,z) of real numbers which satisfy
$$x+y^2+z^2 = x^2+y+z^2 = x^2+y^2+z = a.$$

2. Let P be the point inside a triangle ABC such that
$$\angle PBA = \angle PCA = \frac{1}{3}(\angle ABC + \angle ACB).$$
Prove that $\dfrac{AC}{AB+PC} = \dfrac{AB}{AC+PB}$.

3. Let be given n points, no three of which are on a line. All the segments with endpoints in these points are colored so that two segments with a common endpoint are of different colors. Determine the least number of colors for which this is possible.

Second Day

4. Find all triples of positive integers with the property that the product of any two of them gives the remainder 1 upon division by the third number.

5. We have thrown k white dice and m black dice. Find the probability that the remainder modulo 7 of the sum of the numbers on the white dice is equal to the remainder modulo 7 of the sum of the numbers on the black dice.

6. Let eight points be given in a unit cube. Prove that two of these points are on a distance not greater than 1.

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