25-th Swedish Mathematical Competition 1985

Final Round

November 23, 1985

1. If $a > b > 0$, prove the inequality

$$\frac{(a-b)^2}{8a} < \frac{a+b}{2} - \sqrt{ab} < \frac{(a-b)^2}{8b}.$$

2. Find the least natural number such that if the first digit (in the decimal system) is placed last, the new number is $7/2$ times as large as the original number.

3. Points A, B, C with $AB = BC$ are given on a circle with radius r, and D is a point inside the circle such that the triangle BCD is equilateral. The line AD meets the circle again at E. Show that $DE = r$.

4. Let $p(x)$ be a polynomial of degree n with real coefficients such that $p(x) \geq 0$ for all x. Prove that

$$p(x) + p'(x) + p''(x) + \cdots + p^{(n)}(x) \geq 0.$$

5. In a rectangular coordinate system, O is the origin and $A(a,0)$, $B(0,b)$ and $C(c,d)$ the vertices of a triangle. Prove that

$$AB + BC + CA \geq 2CO.$$

6. X-wich has a vibrant club-life. For every pair of inhabitants there is exactly one club to which they both belong. For every pair of clubs there is exactly one person who is a member of both. No club has fewer than 3 members, and at least one club has 17 members. How many people live in X-wich?

The IMO Compendium Group,
D. Djukić, V. Janković, I. Matić, N. Petrović
www.imomath.com