# 51-th Bulgarian Mathematical Olympiad 2002, IV Round

*First Day, ? april 2002*

1. Let $\{a_n\}_1^\infty$ be a sequence of real numbers, such that $a_{n+1} = \sqrt{a_n^2 + a_n - 1}$. Prove that $a_1 \notin (-2, 1)$.

*Oleg Mushkarov, Nikolai Nikolov*

2. Consider the orthogonal projections of the vertices $A$, $B$ and $C$ of triangle $ABC$ on external bisectors of $\angle ACB$, $\angle BAC$ and $\angle ABC$, respectively. Prove that if $d$ is the diameter of the circumcircle of the triangle, which is formed by the feet of the projections, while $r$ and $p$ are the inradius and the semi-perimetr of $\triangle ABC$, respectively, then $r^2 + p^2 = d^2$.

*Alexander Ivanov*

3. Given are $n^2$ points in the plane, such that no three of them are collinear, where $n \geq 4$ is a positive integer of the form $3k+1$. What is the minimal number of connecting segments among the points, such that for each $n$-plet of points we can find four points, which are all connected to each other?

*Alexander Ivanov, Emil Kolev*

*Second day, ? april 2002*

4. Let $I$ be the incenter of a non-equilateral triangle $ABC$ and $T_1$, $T_2$, $T_3$ be the tangency points of the incircle with the sides $BC$, $CA$, $AB$, respectivey. Prove that the orthocenter of $\triangle T_1T_2T_3$ lies on the line $OI$, where $O$ is the circumcenter of $\triangle ABC$

*Georgi Ganchev*

5. Find all pairs $(b, c)$ of positive integers, such that the sequence definited by

$$a_1 = b, \quad a_2 = c \quad \text{and } a_{n+2} = |3a_{n+1} - 2a_n| \quad \text{for } n \geq 1$$

has only finite number of composite terms.

*Oleg Mushkarov, Nikolai Nikolov*

6. Find the smallest number $k$, such that $\frac{\ell_a + \ell_b}{a+b} < k$ for all triangles with sides $a$ and $b$ and bisectors $\ell_a$ and $\ell_b$ to them, respectively.

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