5-th International Mathematical Olympiad

Wroclaw, Poland, July 5–13, 1963

First Day

1. Determine all real solutions of the equation $\sqrt{x^2-p}+2\sqrt{x^2-1}=x$, where p is a real number. *(Czechoslovakia)*

2. Find the locus of points in space that are vertices of right angles of which one ray passes through a given point and the other intersects a given segment. *(Soviet Union)*

3. Prove that if all the angles of a convex n-gon are equal and the lengths of consecutive edges $a_1,\ldots,a_n$ satisfy $a_1 \geq a_2 \geq \cdots \geq a_n$, then $a_1 = a_2 = \cdots = a_n$. *(Hungary)*

Second Day

4. Find all solutions $x_1,\ldots,x_5$ to the system of equations

$$\begin{cases} x_5+x_2 &= yx_1,\\ x_1+x_3 &= yx_2,\\ x_2+x_4 &= yx_3,\\ x_3+x_5 &= yx_4,\\ x_4+x_1 &= yx_5, \end{cases}$$

where y is a real parameter. *(Soviet Union)*

5. Prove that $\cos\dfrac{\pi}{7}-\cos\dfrac{2\pi}{7}+\cos\dfrac{3\pi}{7}=\dfrac{1}{2}$. *(DR Germany)*

6. Five students A, B, C, D, and E have taken part in a certain competition. Before the competition, two persons X and Y tried to guess the rankings. X thought that the ranking would be A,B,C,D,E; and Y thought that the ranking would be D,A,E,C,B. At the end, it was revealed that X didn't guess correctly any rankings of the participants, and moreover, didn't guess any of the orderings of pairs of consecutive participants. On the other hand, Y guessed the correct rankings of two participants and the correct ordering of two pairs of consecutive participants. Determine the rankings of the competition. *(Hungary)*