# 36-th International Mathematical Olympiad

## Toronto, Canada, July 13–25, 1995

*First Day – July 19*

1. Let $A, B, C$, and $D$ be distinct points on a line, in that order. The circles with diameters $AC$ and $BD$ intersect at $X$ and $Y$. $O$ is an arbitrary point on the line $XY$ but not on $AD$. $CO$ intersects the circle with diameter $AC$ again at $M$, and $BO$ intersects the other circle again at $N$. Prove that the lines $AM$, $DN$, and $XY$ are concurrent. *(Bulgaria)*

2. Let $a$, $b$, and $c$ be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(a+c)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

*(Russia)*

3. Determine all integers $n > 3$ such that there are $n$ points $A_1, A_2, \dots, A_n$ in the plane that satisfy the following two conditions simultaneously:

   (a) No three lie on the same line.

   (b) There exist real numbers $p_1, p_2, \dots, p_n$ such that the area of $\triangle A_i A_j A_k$ is equal to $p_i + p_j + p_k$, for $1 \leq i < j < k \leq n$. *(Czech Republic)*

*Second Day – July 20*

4. The positive real numbers $x_0, x_1, \dots, x_{1995}$ satisfy $x_0 = x_{1995}$ and

$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i}$$

for $i = 1, 2, \dots, 1995$. Find the maximum value that $x_0$ can have.

*(Poland)*

5. Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$, $DE = EF = FA$, and $\measuredangle BCD = \measuredangle EFA = \pi/3$ (that is, $60^\circ$). Let $G$ and $H$ be two points interior to the hexagon, such that angles $AGB$ and $DHE$ are both $2\pi/3$ (that is, $120^\circ$). Prove that $AG + GB + GH + DH + HE \geq CF$.

*(New Zealand)*

6. Let $p$ be an odd prime. Find the number of $p$-element subsets $A$ of $\{1, 2, \dots, 2p\}$ such that the sum of all elements of $A$ is divisible by $p$. *(Poland)*



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